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The allusion is to Ludolph van Ceulen (d. 1610), who computed π to 35 places, and in whose honor the term "Ludolph's number" is current in Germany. The references to Egypt and Archimedes of Syracuse need not be discussed.

Mr. Haldeman shows that the vertices of a regular undecagon inscribed in a given circle lie on a certain sextic curve. The equation of the sextic involves a parameter by variation of which the undecagon may be rotated into all positions on the circle. The method is along the same line as one given in the MONTHLY by the same author in connection with the regular pentagon and the regular heptagon (*1920*, 257–258, *1919*, 390). Some editorial remarks follow the article.

Dr. J. Rosenbaum investigates a problem suggested by the Pythagorean relation. It may be shown that for a triangle ABC with $a > b > c$ there exists a unique $n \geqq 1$ such that $a^n = b^n + c^n$; but only when A is a right angle or a straight angle does n depend solely on A. In general there is a relation connecting A, b/c, and n, which becomes independent of b/c when A is put equal to $\pi/2$ or π. Dr. Rosenbaum shows that, conversely, when A and n are given, this relation is satisfied by at most one real value of b/c, except when $n = 1$ or 2.

The note by Professor Swift calls attention to a fact in connection with trigonometric tables which is something of a paradox, and illustrates the distinction between a transcendental number and a transcendental function.

The failure at the origin of the formula in polar coördinates for the direction of a curve appears to receive no notice in a number of calculus texts. Professor H. J. Ettlinger supplies this want in the short note which forms Discussion V. His result could also be obtained by transforming dy/dx into polar form.

I. UNCLE ZADOCK'S RULE FOR OBTAINING THE DOMINICAL LETTER FOR ANY YEAR.[1]

By W. H. VAIL, Newark, N. J.

All of our measurements of time are either natural or conventional in character. Thus the solar year, the solar day, and the lunar month are natural measurements of time, being governed by the movements of the sun, the moon, and the earth in their various orbits; while all the other measurements of time, such as the second, the minute, the hour, the week, the civil month, the civil year, and the century, are conventional or arbitrary in character, having been formed for the convenience of man.

All almanacs and diaries which contain the Church Calendar, and the list of eclipses, with the chronological cycles, will cite among other items the Dominical letter for the year which the diary or almanac represents. Thus for the year 1921 they will state that the Dominical letter is B, which is pure Greek to almost every person who consults it. The first problem which the construction of a calendar presents is to find the day of the week corresponding to a given day of the year. The week contains seven days, and the problem would be perfectly simple if the year was composed of a whole number of weeks; but the ordinary year contains fifty-two weeks and one day, while the leap year contains fifty-two weeks and two days.

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[1] An explanation of the Dominical Letter with tables is given in *The Calendar, its History, Structure and Improvement*, by Alexander Philip, Cambridge University Press, 1921.

In order to facilitate the solution of this problem, the days of the week are denoted by the first seven letters of the alphabet, *A, B, C, D, E, F, G,* which are placed for convenience opposite the days of the year; thus beginning with *A* standing opposite the first day of January, *B* stands opposite the second, *G* opposite the seventh, then *A* opposite the eighth, and so on throughout the year. Following this rule it is found that *A* will represent January 1, 8, 15, 22, and 29, which would bring *B* to represent January 30 and *C* January 31. Then *D* will represent February 1, and as February in the ordinary year contains only twenty-eight days, *D* will also represent March 1, 8, 15, 22, and 29, bringing *E* for March 30, *F* for March 31, and *G* for April 1. In this connection it may be remarked that the letter for each month is fixed. That is, *A* is always the letter for the first day of January, *D* for the first day of February and of March, whether the year be an ordinary year or a leap year, *G* the letter for April, and so on.

To aid the memory in remembering which letters represent the different months of the year, the following couplet has been formed: At Dover Dwell George Brown Esquire, Good Carlos Finch, And David Friar. The initial letters of the above twelve words in their order stand for the first days of the twelve months of the year. Or you may take the twelve letters themselves in their regular order, thus: *ADD G, BEG C, FAD F.* A little practice will soon associate the individual letters with the months they represent.

Now proceeding a step further, if one of the days of the week, Sunday for example (as is the case with the year 1921), is represented by the letter *B*, Monday would be represented by *C*, Tuesday by *D*, Wednesday by *E*, Thursday by *F*, Friday by *G*, and Saturday by *A*, returning for Sunday to *B* again; every Sunday would be represented by the same letter, *B*, through the year. If we are able to obtain the letter which represents Sunday for the year, we know in this way the letters which represent the other days of the week, with a special provision for leap years. Having established the letters which shall stand for the first days of the months, it only remains to obtain the Sunday letter for the year. The letter which represents Sunday for the year is called the Dominical or Sunday letter and when this is obtained for any year, the letters which denote the first days of the various months tell us the days of the week upon which these first days will fall. Thus *B* representing Sunday for the year 1921 (as you will find so stated in any almanac) and *B* being the letter which represents May 1, it follows that May 1, 1921, occurs on Sunday. *C* being the letter which represents the first of August, it follows that August 1, 1921, occurs on Monday, and so on through the seven letters which represent the first days of the various months for the year 1921. Thus we are able very readily to state upon what day of the week the first day of any month occurs, and knowing this the days of the week upon which the other days of the month occur follow automatically after a little practice.

This naturally brings us to the consideration of the subject proper of this paper which is Uncle Zadock's rule for obtaining the Dominical or Sunday letter for any year. Some fourscore years ago there appeared an article in the *New York Observer* entitled "A rule for obtaining the Dominical letter for any year." This rule was originated by a school teacher in Pennsylvania whose real name was concealed under the pseudonym of "Uncle Zadock." Therefore the rule came to be known as "Uncle Zadock's Rule." As far as we are aware the real name of the author of this rule was never revealed, and during all these years we have seen it in print only once, when the writer's father had it inserted in a local paper in northern New Jersey, where he resided. This rule was given without proof. At the same time it is absolutely correct in its application to any date in the past, present or future of the Christian Era.[1] It is also most simple in its operation; and if any person is inclined to question its simplicity, let him scan the rule given in the *Encyclopædia Britannica* and wade through the mazes of figures and unknown quantities, as they occupy page after page of that work, before being able to obtain the Dominical letter for any year in question.

In contrast let us now consider Uncle Zadock's rule, and learn how readily the Dominical or Sunday letter for any year is obtained. The rule is as follows:

First, divide the centurial number of the year in question by four.

Second, multiply the remainder, if any, by two.

Third, subtract the product from six.

Fourth, to the remainder add the odd years and also the fourth part of the odd years, rejecting fractions.

Fifth, divide the sum of these numbers by seven.

Sixth, subtract the remainder, if any, from seven.

The last remainder indicates the Dominical or Sunday letter for the year under consideration in the order of the first seven letters of the alphabet. Thus should the last remainder be 1, the Dominical letter would be *A*. If the remainder were 2, the letter would be *B*, and so on.

Should the year under consideration prove to be a leap year, the Dominical or Sunday letter obtained by this rule would apply to the last ten months of the year, and the Sunday letter for the months of January and February in that year would be the next letter in the order of the first seven letters of the alphabet. Thus for the year 1920, by this rule, you would obtain the Dominical letter as *C* and this would be the Dominical letter for the last ten months of the year. The Sunday letter for January and February of this year would be the next letter, which is *D*.

As an example under the above rule, let us see upon what day of the week the fourth of July, 1776, fell. To obtain the Dominical letter for 1776, divide the centurial number 17 by 4, and you have, as a remainder, 1. Twice this remainder is 2, 2 from 6 leaves 4, and 4 added to the odd number of years, 76, and to one fourth of 76, which is 19, gives 99. Divide 99 by 7 and subtract the remainder 1 from 7. The final remainder 6 represents the sixth letter of the alphabet which is *F*; and therefore *F* is the Dominical or Sunday letter for the last ten months of the year 1776; for 1776 was a leap year, and Uncle Zadock's rule always gives you the Sunday letter for the last ten months of leap years. *F* being the Sunday letter for the last ten months of the year, and the letter for July always being *G*, it follows that July 1, 1776, fell on Monday and July 4 on Thursday.

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[1] The direct application of the rule holds only for the Gregorian Calendar or "New Style" dating; "Old Style" dates must first be converted into "New Style," as is readily done.

We may note that within a given century the first three steps in the process will always yield the same result. Thus in the nineteenth century there will always be two and in the twentieth century nothing to be added to the odd numbers and the fourth part of them. In applying this rule to any date within the present century, we may simply ignore the centurial number entirely and proceed with the fourth step in the rule.

II. CONSTRUCTION OF THE REGULAR UNDECAGON BY A SEXTIC CURVE.

By CYRUS B. HALDEMAN, Ross, Butler County, Ohio.

Consider the sextic

$$x(a^2 + 3ay^2 + y^4)\sqrt{-(b^2 + a^{11})} = a^5y^6 - by^5 + 6a^6y^4 - 5aby^3 + 9a^7y^2 - 5a^2by + 2a^8, \tag{1}$$

which will be real when $b^2 + a^{11}$ is negative, and the circle

$$x^2 + y^2 = -4a, \tag{2}$$

which will be real when a is negative.

Eliminate x from (1) by means of (2). After expansion, reduction, and separation into factors, the result may be written in the form

$$(y^{11} + 11ay^9 + 44a^2y^7 + 77a^3y^5 + 55a^4y^3 + 11a^5y + 2b)(a^5y - 2b) = 0.$$

By the transformation $y = -2s\sqrt{-a}$, the first of these factors placed equal to zero gives the equation

$$11s - 220s^3 + 1232s^5 - 2816s^7 + 2816s^9 - 1024s^{11} = \frac{b}{a^5\sqrt{-a}};$$

and because

$$11\sin A - 220\sin^3 A + 1232\sin^5 A - 2816\sin^7 A + 2816\sin^9 A - 1024\sin^{11} A = \sin 11A,$$

we may take

$$s = \sin A, \qquad \text{where} \qquad \sin 11A = \frac{b}{a^5\sqrt{-a}},$$

and get

$$y = -2\sqrt{-a}\sin\frac{1}{11}\sin^{-1}\frac{b}{a^5\sqrt{-a}},$$